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Spiral Vortices and Liquid Breakup

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Introduction

INTERNAL cavity flows occur in many engineering environments, including the internal flow in solid-propellant rocket motors. Liquids occur in the metalized solid-propellant rocket motor environment in the form of molten aluminum or aluminum oxide droplets.[1] This Note reports on the observation of axially directed spiral vortices in an internal cavity flow and the breakup of a liquid droplet placed in the initiation point of a vortex. With the occurrence of axially directed spiral vortices from wall shear layers[2] and from separated free shear layers as indicated in this Note, a dynamic process is identified that could provide a key mechanism for breaking liquid aluminum and aluminum oxide droplets and agglomerates from propellant surfaces in aluminized solid-propellant rocket motors.

Experimental Facilities and Results

This study was carried out in the Subsonic Turbulent Flow Facility in the Department of Mechanical and Industrial Engineering at the University of Utah. This facility and the internal flow cavity employed in the study are described in Ref. 3.

The flow facility has a test section 2.44 m in length and a cross section 177.8 mm on a side. The first 0.61 m of the top surface of the test section is covered with a medium coarse sandpaper to promote transition of the boundary layer to a turbulent flow state. A schematic of the internal flow cavity is shown in Fig. 1, with the location of the free shear layers indicated in the diagram. Velocity profiles were obtained with an X-configuration, constant-temperature, hot-film anemometer system with linearized output signals. Mean velocity values were obtained as the average of 50 samples at each vertical location in the flow region. The axial velocity profile shown in Fig. 2 was obtained with a traversing mechanism driven with a computer-controlled stepper motor with 1,000 vertical steps in increments of 0.08 mm, for a total distance of 80 mm across the center of the flow region. This profile was obtained at an axial distance of 2 mm downstream of the edge of the forward restrictors. The Reynolds shear stress profile shown in Fig. 3 was obtained with a Honeywell model 9410 correlator for this same axial station.

The Reynolds shear stress was obtained with a boundary-layer type of probe with X-configuration sensors. The probe was inserted from the bottom wall of the cavity with the sensors intersecting the free shear layer velocity gradient from below. Probe-induced flow oscillations should be minimized with this procedure.[4]

The Reynolds shear stress profile indicates several significant aspects of the flow in the upper and lower free shear layers. Note that the upstream turbulent nature of the incoming boundary layer has a dramatic effect on the Reynolds shear stress in the upper free shear layer, as previously indicated by Hussain and Clark,[5] with a large component of velocity fluctuations carried into the free shear layer from the upstream turbulent boundary layer along the top wall of the test section.

The Reynolds shear stress profile through the lower free shear layer indicates the presence of four counterrotating, axially directed vortices. The possible existence of the axial vortex structures was noted initially by the discovery of high-frequency, high-intensity velocity fluctuations ("turbulent bursts") at a distance of approximately 2 mm downstream and 2 mm above the edge of the lower forward restrictor. These bursts were not observed forward of a station 1 mm downstream and 2 mm above the edge of the lower restrictor. These bursts are similar to the bursts observed by Lindgren,[6] as reported by Joseph,[7] and are also similar to the bursts observed in wall shear layers by Blackwelder and Eckelmann[2] and Maslowe.[8]

In these experiments, we are primarily interested in identifying the interaction of counterrotating pairs of vortices and liquid droplets. Water is injected into the flow stream with a 0.5 mm needle with the liquid injection point placed in the mainstream flow and the liquid allowed to run down the needle until it is held in place at the critical layer in the shear layer. The droplet is held in place by the vertical velocity component induced by the lower pair of counterrotating vortices. The droplet, which forms on the downstream side of the needle, is observed to oscillate and then to separate into multiple droplets, with several droplets swept from the supporting needle surface. A portion of the orginal droplet remains on the surface of the needle. Figure 4 shows several larger droplets with unstable ligaments of fluid between them. These ligaments break up into smaller droplets. Downstream of the injection point, the larger droplets swept from the supporting needle deform into a parachute-like shape prior to breakup due to aerodynamic effects. Note that the injection needle was placed at the entrance to the flow cavity and that the flow is from right to left in the photograph of Fig. 4.

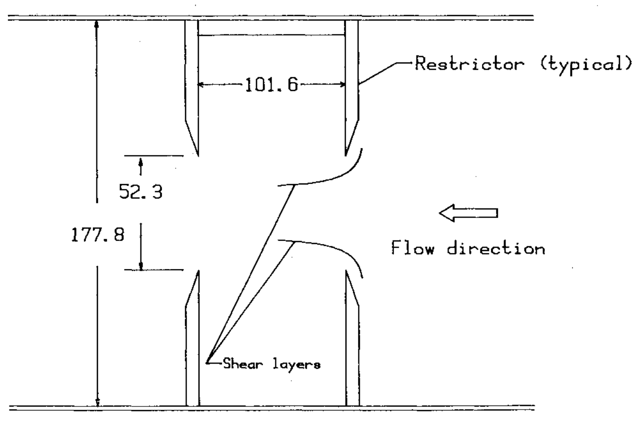

Fig. 1 Schematic diagram of the internal flow cavity showing the curved shear layers in the forward region of the cavity.

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The injection needle should not be the source of the axially directed vortex structures, since the vortex structures were observed without the needle in place. The injection needle may, however, promote the merger of the vortex structures produced along the curved flow separation line into larger, more dynamic, and unsteady spiral vortices. A spiral vortex will possess a significant internal axial pressure gradient,

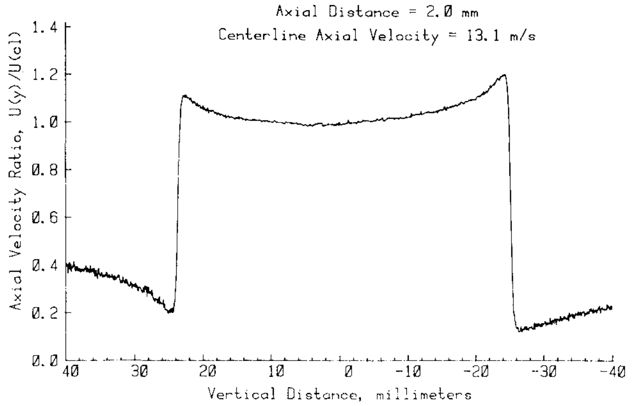

Fig. 2 Profile of the ratio of axial velocity to the centerline velocity at an axial station 2 mm downstream of the forward set of restrictors for the internal cavity flow shown in Fig. 1.

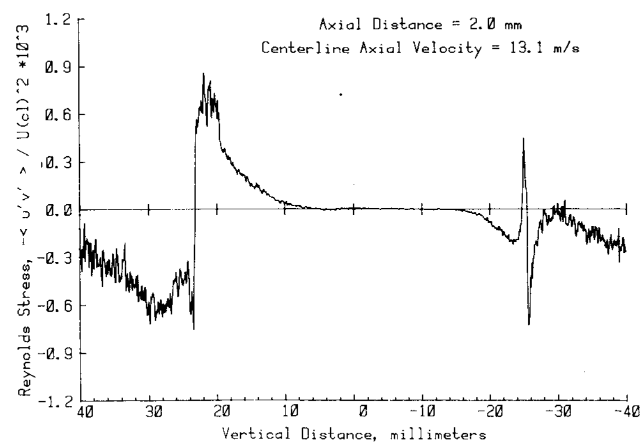

Fig. 3 Profile of the dimensionless Reynolds shear stress across the flow region shown in Fig. 1 at an axial station 2 mm downstream of the forward set of restrictors. Note the indication of two pairs of counterrotating longitudinal vortices at a vertical station of − 24 mm from the center axis of the flow.

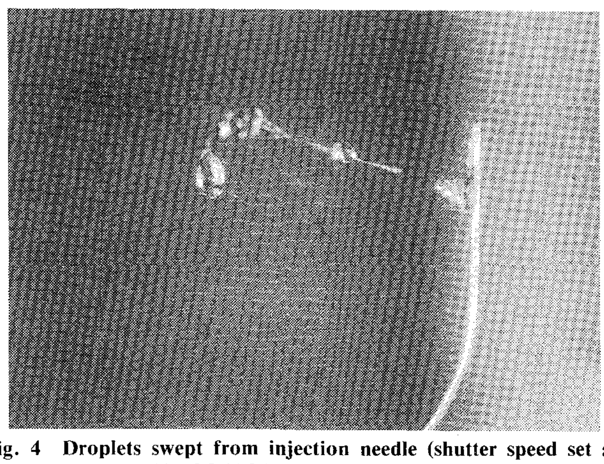

Fig. 4 Droplets swept from injection needle (shutter speed set at 1/60 s, aperture f/4, ISO/ASA 200, with two reflected flood lamps and two reflected flash units for illumination).

which will apply a force to the droplet in the axial direction, as well as vorticity frequencies of the same order of magnitude as the surface resonance frequencies of the droplet. Hence, two interactive mechanisms exist to produce droplet separation from the supporting injection needle. Neither the irrotational mainstream flow nor the shear layer velocity profile is sufficiently large in magnitude to cause the observed separation and breakup. The critieria established by Acrivos[9] applied to the flow conditions in this experiment provide the basis for these observations.

With the presence of axially directed vortex structures erupting from the burning propellant surface of aluminized solid-propellant rocket motors, a strong mechanism exists for removal and breakup of molten aluminum droplets and agglomerates into smaller droplets within the combustion chamber. With the development of internal reactive viscous flow analysis procedures based upon the local mean turbulent kinetic energy and the local mean rate of dissipation of turbulent kinetic energy,[10,11] a format exists that can be extended to include dynamic vortex dissipation processes in the turbulent flow analysis. Cantwell[12] has reviewed experimental measurements of instantantous values of local Reynolds stress $u'v'$ in wall shear layers, with values 10-60 times the local mean values of the Reynolds shear stress. The identification of a significant dynamic process produced by dissipating vortex structures will allow extension of the analysis procedure to the agglomerating aluminum breakup processes.[13,14] This project is studying the relationship between the local instantantous dissipation rate in isolated vortex structures and the mean rate of dissipation of turbulent kinetic energy for inclusion in an appropriate analysis procedure. Small-scale windowed aluminized solid-propellant rocket motor tests with appropriate cross-flow velocities are required to verify the importance of these vortices in molten aluminum breakup.

As indicated in the schematic diagram in Fig. 1, the internal free shear layers are curved, with the flow converging into the internal cavity and accelerating to the longitudinal center of the cavity. Further downstream, the flow diverges and decelerates to the exit plane of the cavity. Considerable literature has grown up around the study of Görtler vortices[15-17] formed from instabilities in the boundary layers along concave surfaces. The shear layers occurring in the internal cavity flow here may conform more to the configuration analyzed by Witting,[18,19] which predicts pairs of counterrotating axially directed spiral vortices existing along convex curved free shear layers. These previous studies, together with the recent work of Lundgren,[20] should form a basis for the analysis of internally generated vortex structures for application to the prediction of droplet breakup in solid-propellant rocket motors.

Conclusions

Measurement of mean Reynolds stress in the near field of an internal free shear layer has indicated the presence of pairs of counterrotating axially directed vortices. The interaction of these spiral vortices with suspended water droplets produces droplet breakup and acceleration into the flow stream. Smaller droplets are produced by breakup of the connecting filament of fluid between the separating larger droplets.

Acknowledgments

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Mean Square Response to Band-Limited White Noise Excitation

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Introduction

THE stationary response of a stable time-invariant linear multidegree-of-freedom (MDF) system to white noise and second-order filtered white noise excitation was studied by the authors[1,2] via complex modal anlaysis. However, an important theoretical (as well as practical) case, i.e, that for band-limited white noise excitation, has not been addressed. An exact solution for the mean square response of a single degree-of-freedom system to band-limited white noise was given by Crandall and Mark[3] about 20 years ago. Since then its counterpart for MDF systems has not appeared. However, an exact solution for MDF systems based on Ref. 1 is given here.

The System Response

The system considered is time-invariant linear MDF, either classically damped or not and symmetrical or not. Its differential equation can be described as follows:

$$m\ddot{y}+c\dot{y}+ky=f(t) \tag{1}$$

where y is the system response; m, c, and k the mass, damping, and stiffness matrices, respectively; and $f(t)$ the band-limited white noise excitation with zero mean and the following autospectral matrix:

$$S_f(\omega)=D, \quad \text{when } \omega_1<|\omega|<\omega_2$$
$$=0, \quad \text{elsewhere} \tag{2}$$

where D is a real symmetrical non-negative constant matrix.

The system response may be found in a somewhat indirect way. First, we find the system response x to white noise excitation $w(t)$ with zero mean and the autospectral matrix

$$S_w(\omega)=D$$

Its correlation function matrix is

$$R_w(\tau)=2\pi D\delta(\tau)$$

In this case, the differential equation is

$$m\ddot{x}+c\dot{x}+kx=w(t) \tag{3}$$

When the damping is below critical, all of the system eigenvalues appear as complex conjugates with negative real part

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